

**Quiz 1****Question 1. (12 pts)**

- (a) (6 pts) Find equations of the line  $L$  that passes through the point  $A(1, 0, 4, 3)$  and is perpendicular to the plane  $x_1 + x_2 + x_3 + x_4 = 1$ .

**Solution:** The direction of the line is

$$(1, 1, 1, 1)$$

the equations of the line are

$$\begin{cases} x_1 = t + 1 \\ x_2 = t \\ x_3 = t + 4 \\ x_4 = t + 3 \end{cases}$$

- (b) (6 pts) Find a *unit* vector that is orthogonal to both vectors  $v = (1, 0, 2)$  and  $u = (0, 1, 3)$ .

**Solution:** We need a normal vector of the hyperplane.

$$n = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{vmatrix} = (-2, -3, 1)$$

Now scale it to a unit vector

$$v = \left( \frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right)$$

**Question 2. (8 pts)**

Find an equation of the plane  $H$  that passes through the points  $A(1, 0, 1)$ ,  $B(0, 2, 0)$  and  $C(1, 2, 2)$ .

**Solution:** Consider

$$\overrightarrow{AB} = (-1, 2, -1)$$

$$\overrightarrow{AC} = (0, 2, 1)$$

then a norm vector of  $H$  is

$$n = \begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix} = (4, 1, -2)$$

So the equation of  $H$  takes the form

$$4x + y - 2z = k$$

plug in, say,  $A(1, 0, 1)$  and we get

$$k = 2$$

So the plane  $H$  is

$$4x + y - 2z = 2$$